Advances in Stochastic Geometry for Cellular Networks

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Final Presentation for the degree of
Doctor of Philosophy
in
Electrical Engineering

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Shyam Ranganathan

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Overview of Research Contributions

- Thrust-I: 3GPP-inspired HetNet models
- Thrust-II: Mm-wave integrated access and backhaul network models
- Thrust-III: Machine learning meets stochastic geometry

Publications (1)

Book

[B1] H. S. Dhillon, **C. Saha**, and M. Afshang, *Poisson Cluster Processes: Theory and Application to Wireless Networks*, under preparation. Cambridge University Press, 2021. Repository: https://github.com/stochastic-geometry/PCP-Book.

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Journals

[J10] **C. Saha**, M. Afshang, and H. S. Dhillon, "Meta Distribution of Downlink SIR in a Poisson Cluster Process-based HetNet Model", under revision, *IEEE Wireless Commun. Letters*. Available online: arxiv.org/abs/2007.05997.

[J9] **C. Saha**, H. S. Dhillon, "Load on the Typical Poisson Voronoi Cell with Clustered User Distribution", *IEEE Wireless Commun. Letters*, to appear.

[J8] **C. Saha**, H. S. Dhillon, "Millimeter Wave Integrated Access and Backhaul in 5G: Performance Analysis and Design Insights ", *IEEE Journal on Sel. Areas in Commun.*, vol. 37, no. 12, pp. 2669-2684, Dec. 2019.

[J7] **C. Saha**, H. S. Dhillon, N. Miyoshi, and J. G. Andrews, "Unified Analysis of HetNets using Poisson Cluster Process under Max-Power Association", *IEEE Trans. on Wireless Commun.*, vol. 18, no. 8, pp. 3797-3812, Aug. 2019.

Publications (2)

[J6] **C. Saha**, M. Afshang, and H. S. Dhillon, "Bandwidth Partitioning and Downlink Analysis in Millimeter Wave Integrated Access and Backhaul for 5G", *IEEE Trans. Wireless Commun.*, vol. 17, no. 12, pp. 8195-8210, Dec. 2018.

[J5] M. Afshang, **C. Saha**, and H. S. Dhillon, "Equi-Coverage Contours in Cellular Networks", *IEEE Wireless Commun. Letters*, vol. 7, no. 5, pp. 700-703, Oct. 2018.

[J4] **C. Saha**, M. Afshang, and H. S. Dhillon, "3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage", *IEEE Trans. on Commun.*, vol. 66, no. 5, pp. 2219-2234, May 2018.

[J3] M. Afshang, **C. Saha**, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process", *IEEE Commun. Letters*, vol. 21, no. 12, pp. 2686-2689, Dec. 2017.

[J2] **C. Saha**, M. Afshang, and H. S. Dhillon, "Enriched K-Tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments", *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 1593-1608, Mar. 2017.

[J1] M. Afshang, **C. Saha**, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process", *IEEE Wireless Commun. Letters*, vol. 6, no. 1, pp. 130-133, Feb. 2017.

Publications (3)

Conference Proceedings

[C7] **C. Saha** and H. S. Dhillon, "Interference Characterization in Wireless Networks: A Determinantal Learning Approach", in Proc. IEEE Int. Workshop in Machine Learning for Sig. Processing, Pittsburgh, PA, Oct. 2019.

[C6] **C. Saha** and H. S. Dhillon, "Machine Learning meets Stochastic Geometry: Determinantal Subset Selection for Wireless Networks", in Proc. IEEE Globecom, Waikoloa, HI, Dec. 2019.

[C5] **C. Saha** and H. S. Dhillon, "On Load Balancing in Millimeter Wave HetNets with Integrated Access and Backhaul", in Proc. IEEE Globecom, Waikoloa, HI, Dec. 2019.

[C4] **C. Saha**, M. Afshang and H. S. Dhillon, "Integrated mmWave Access and Backhaul in 5G: Bandwidth Partitioning and Downlink Analysis", in Proc. IEEE ICC, Kansas City, MO, May 2018.

Publications (4)

[C3] **C. Saha**, M. Afshang and H. S. Dhillon, "Poisson Cluster Process: Bridging the Gap Between PPP and 3GPP HetNet Models", in Proc. ITA, San Diego, CA, Feb. 2017.

[C2] **C. Saha** and H. S. Dhillon, "D2D Underlaid Cellular Networks with User Clusters: Load Balancing and Downlink Rate Analysis", in Proc. IEEE WCNC, San Francisco, CA, March 2017.

[C1] **C. Saha** and H. S. Dhillon, "Downlink Coverage Probability of K-Tier HetNets with General Non-Uniform User Distributions", in Proc. IEEE ICC, Kuala Lumpur, Malaysia, May 2016.

Other publications

[C9] K. Bhogi, **C. Saha**, H. S. Dhillon, "Learning on a Grassmann Manifold: CSI Quantization for Massive MIMO Systems", available online: arxiv.org/abs/2005.08413.

[C8] S. Mulleti, **C. Saha**, H. S. Dhillon, Y. Elder, "A Fast-Learning Sparse Antenna Array", in Proc. IEEE RadarConf20, to appear.

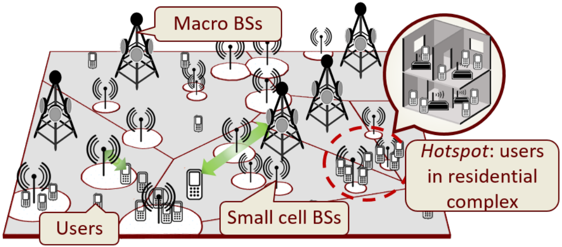
Section I

Introduction

Stochastic Geometry | Random Spatial Models of Cellular Networks | Poisson Point Process Models | Spatial Couplings

Modeling and Analysis of Cellular Networks

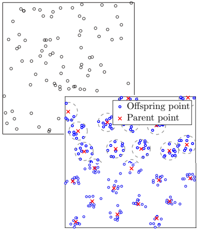
- A heterogeneous cellular network (HetNet) consists of a variety of base stations (BSs) such as macro BSs (MBSs) and small cell BSs (SBSs) and users (e.g. pedestrians, hotspots).
- Performance characterization:
  - Extensive system-level simulations
  - Simulation settings standardized by 3GPP



- Analytical performance characterization: Stochastic geometry
  - Assumes locations of BSs and users as realization of point processes.

Point Processes

A point process $\Phi = \{x_1, x_2, \dots\}$ is a random sequence of points in $\mathbb{R}^2$ (in general $\mathbb{R}^d$).



Example of point processes

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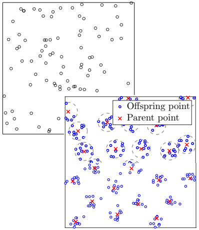

Example of point processes

Measure formalism. A point process is a random counting measure.

- If $B \subset \mathbb{R}^2$ is a Borel set, $\Phi(B) = \#$ of points of Φ in B is a random variable.
- More formally Φ is a measurable mapping from a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ to the counting measure space.

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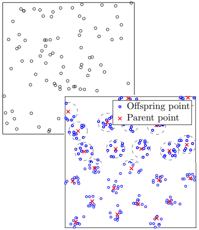

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Measure formalism. A point process is a random counting measure.

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- More formally Φ is a measurable mapping from a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ to the counting measure space.

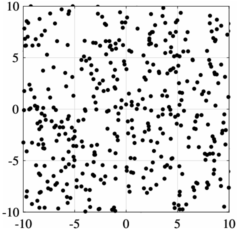
How to specify the distribution of Φ?

- Moments: $\mathbb{E}\left[\left(\sum_{x\in\Phi\cap B} f(x)\right)^n\right]$
- Probability generating functional (PGFL): $G_\Phi[v] := \mathbb{E}\left[\prod_{x\in\Phi} v(x)\right]$
- Sum product functional (SPFL): $S_\Phi[g, v] := \mathbb{E}\left[\sum_{x\in\Phi} g(x) \prod_{y\in\Phi} v(y)\right]$
- Void probability: $\mathbb{P}(\Phi(B) = 0)$ for some closed Borel set B.

Poisson point process (PPP)

Φ is a PPP with intensity measure Λ if

- $\Phi(B) \sim \texttt{Poisson}(\Lambda(B))$
- $\Phi(B_1), \Phi(B_2)$ are independent if B_1 and B_2 are disjoint
- When $\Lambda(B) = \lambda \text{vol}(B)$, Φ is a homogeneous PPP.
- Roughly speaking, PPP places points uniformly at random independently of each other.

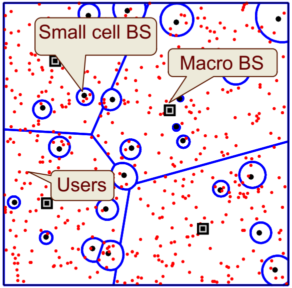


Realization of homogeneous PPP with $\lambda = 1 \text{ m}^{-2}$.

First Comprehensive Model (*PPP Model*)

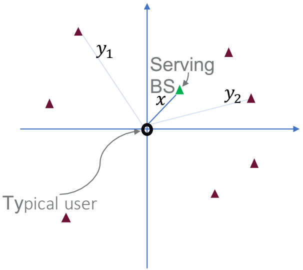
Model different types of BSs and users as independent Poisson Point Processes (PPPs)*.

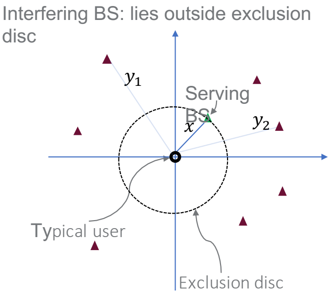
- BS locations of the k-th tier are modeled as a homogeneous PPP $\Phi^{(k)}$ of density λ_k.
- Model users as a PPP ($\Phi_{\rm u}$) independent of the BSs.
- Assumes independence across everything.
- Very tractable (next slide).



* [Dhillon2012] H. S. Dhillon, R. K. Ganti, F. Baccelli and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," IEEE JSAC, 2012.

Why is a PPP model tractable?





Interfering BSs outside the exclusion disc are still PPP.

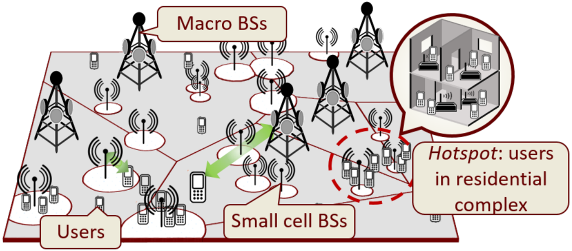
$$\mathbb{P}(\mathtt{SINR} > \beta) \text{ where } \mathtt{SINR} = \frac{\overbrace{P h_{x^*} \|x^*\|^{-\alpha}}^{\text{signal power}}}{\underbrace{\mathtt{N}_0}_{\text{noise power}} + \underbrace{\sum_{x \in \Phi \setminus \{x^*\}} P h_x \|x\|^{-\alpha}}_{\text{interference}}}.$$

Assuming Rayleigh fading, *, $\mathrm{P}_{\mathrm{cPPP}} = \frac{\pi}{C(\alpha)} \frac{\sum_{i=1}^{K} \lambda_i P_i^{\frac{2}{\alpha}} \beta_i^{-\frac{2}{\alpha}}}{\sum_{i=1}^{K} \lambda_i P_i^{\frac{2}{\alpha}}}$.

*"Modeling and analysis of K-tier downlink heterogeneous cellular networks," IEEE JSAC, 2012.

PPP Model: How "accurate" it is?

Key features missing in this "baseline" PPP model:



- Non-uniformity in the user distribution
  - Modeling all users as an independent PPP is not realistic.
  - Fraction of users form spatial clusters (Hotspots), e.g. users in public places and residential areas.
- Correlation between small cell BS (SBS) and user locations
  - Operators deploy SBSs (e.g., picocells) at the areas of high user density.
- Inter and intra BS-tier dependence:
  - BS locations are not necessarily independent.
  - Site planning for deploying BSs introduces correlation in BS locations.

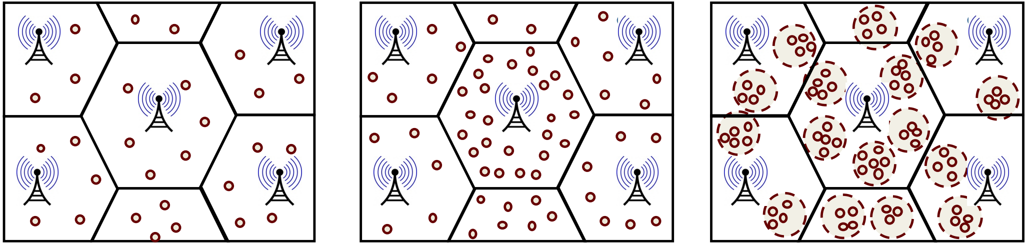
Section II

3GPP Models of HetNets

Existing standards for spatial configurations of users and base stations in network simulation

3GPP Models: User Distributions

3GPP considers different configurations of SBSs and users in HetNet simulation models [†] .



Users “uniform” across macro cells

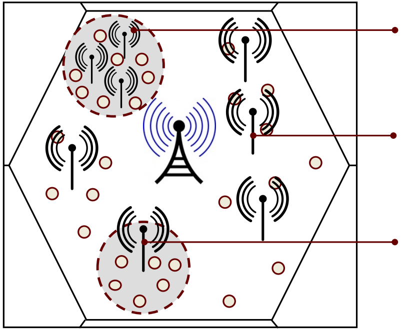
Users “non-uniform” across macro cells

Users forming clusters within a disc

User configurations usually considered in 3GPP HetNet models.

[†] 3GPP TR 36.932 V13.0.0 , “3rd generation partnership project; technical specification group radio access network; scenarios and requirements for small cell enhancements for E-UTRA and E-UTRAN (release 13),” Tech. Rep., Dec. 2015.

3GPP Model: SBS Distribution



SBS Configurations in 3GPP HetNet Model.

Thinking beyond PPP: Is there something that is almost as tractable as a PPP but could model these other morphologies accurately?

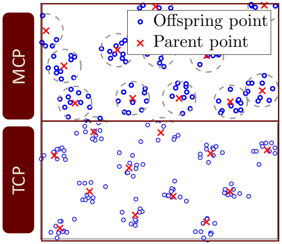
The Answer is Poisson Cluster Process

Poisson Cluster Process (PCP) is more appropriate abstraction for user and BS distributions considered by 3GPP.

Definition (PCP)

A PCP is generated from a PPP $\Phi_{\rm p}$ called the **Parent PPP**, by replacing each point $\mathbf{z}_i$ by a finite offspring point process $\mathcal{B}_i$ where each point is independently and identically distributed around origin.

$$\Phi = \bigcup_{\mathbf{z}_i \in \Phi_{\rm p}} \mathbf{z}_i + \mathcal{B}_i.$$



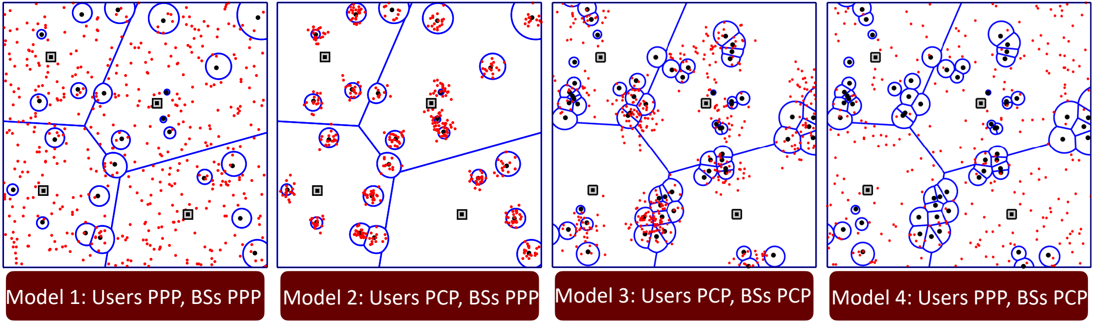
Examples of PCPs

Special Cases

- $\mathcal{B}_i(\mathbb{R}^2) \sim \texttt{Poisson}(\bar{m})$.
- **Matérn Cluster Process**: Each point in $\mathcal{B}_i$ is uniformly distributed inside disc of radius $\texttt{R}$ centered at origin.
- **Thomas Cluster Process**: Each point in $\mathcal{B}_i$ is normally distributed (with variance σ^2) around the origin.

PCP-based Canonical HetNet Models

New HetNet models generated by combining PCPs with PPPs have closer resemblance with 3GPP HetNet models‡.



‡Model 1: [Dhillon2012]
Model 2: [Saha2017] Saha *et. al.* "Enriched K-tier HetNet Model to Enable the Analysis of User-Centric Small Cell Deployments", *IEEE TWC*, 2017.
Models 3,4: [Saha2019] Saha *et. al.*,"Unified Analysis of HetNets using Poisson Cluster Process under Max-Power Association", *IEEE TWC*, 2019.
Unified: [Saha2018] Saha *et. al.*, ., "3GPP-inspired HetNet Model using Poisson Cluster Process: Sum-product Functionals and Downlink Coverage", *IEEE TCOMM*, 2018.

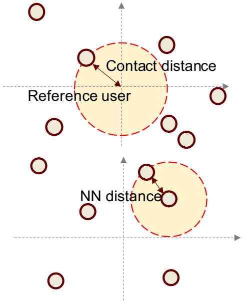
Section III

PCP-based General HetNet Models

PPP and PCP as BS and user distributions | Coverage Analysis

PCP Fundamentals: Distance Distributions

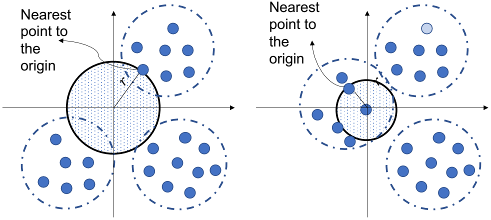
- The first step in the analysis under max-power association is to find the distribution of the distance from the typical user to the BS providing the maximum received power.
- Mathematically, this corresponds to determining nearest-neighbor and/or contact distance distribution of the point process.



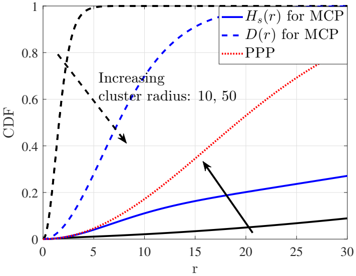
- For PPP, these distributions are the same and are $\mathrm{Rayleigh}((2\pi\lambda)^{-1})$.
- For PCP, these distributions were not well-studied. We have characterized these distributions for TCP and MCP[§].

[§] M. Afshang, C. Saha, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Thomas Cluster Process", *IEEE Wireless Commun. Letters*, 2017.
M. Afshang, C. Saha, and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process", *IEEE Commun. Letters*, 2017.

Contact and Nearest Distance Distributions of PCP



Contact distance   Nearest neighbor distance



CDF of contact and nearest distance distribution of MCP in $\mathbb{R}^2$. For the MCP, $\lambda_{\rm p} = 20 \times 10^{-6}, \bar{m} = 30$.

- For contact distance, we need the void probability of PCP: $\mathbb{P}(\Phi(b(0,r)) = 0)$.
- For nearest neighbor distance, we need

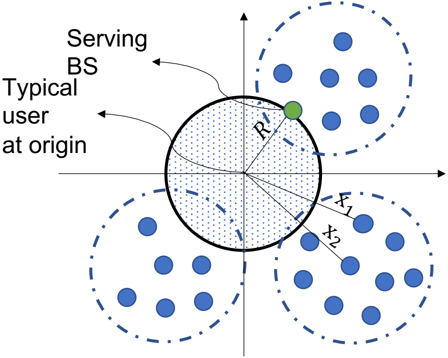
$$\mathbb{P}(\Phi(b(0,r)) \setminus \{o\} = 0 | o \in \Phi) = \underbrace{\mathbb{P}^{!o}(\Phi(b(0,r)) = 0)}_{\text{Reduced Palm measure}},$$

i.e. the void probability of PCP under its reduced Palm measure.

The nearest-neighbor and contact distance distributions of PCP significantly differ PPP.

Coverage Analysis when BSs are PCP: A Toy Example

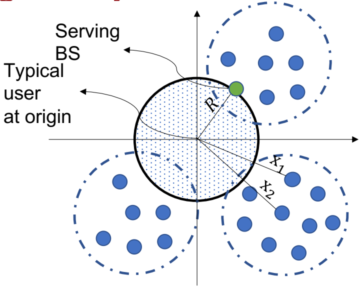
Consider a single tier network where BSs are distributed as PCP. The typical user is located at the origin (no user BS coupling).



R: Serving link distance
$x_1, x_2, \ldots$: Interfering link distance

We assume the small scale fading on each link is i.i.d. Rayleigh, i.e. $h_x \overset{\text{i.i.d.}}{\sim} \exp(1)$.

Coverage Analysis when BSs are PCP: Challenges



R: Serving link distance
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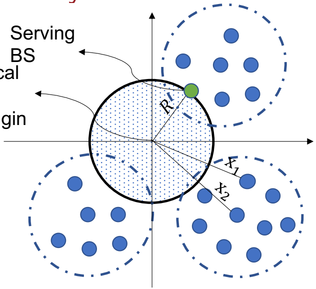
Assuming interference-limited network,

$$\mathrm{P_c} = \mathbb{P}\Bigg(\frac{h_{x^*}\overbrace{\|x^*\|^{-\alpha}}^{\text{contact distance}, R}}{\sum_{x\in\Phi\setminus\{x^*\}} h_x\|x\|^{-\alpha}} > \beta\Bigg)$$

$$= \mathbb{E}\Bigg[\prod_{x\in\Phi\cap b(0,R)} \frac{1}{1+\beta\frac{\|x\|^{\alpha}}{R^{\alpha}}}\Bigg]$$

Coverage Analysis when BSs are PCP: Challenges



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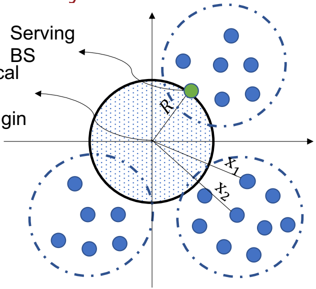
Challenge: Conditioned on R, the distribution of Φ outside the disc $b(0, R)$ is not known.

Coverage Analysis when BSs are PCP: Challenges



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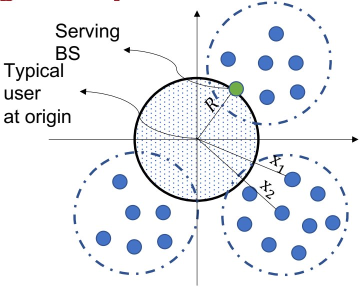
Challenge: Conditioned on R, the distribution of Φ outside the disc $b(0,R)$ is not known.
Solution: Conditioned on the locations of the points of the parent PPP Φ_p, a PCP $\Phi(\lambda_\mathrm{p}, \bar{m}, f)$ is a PPP in $\mathbb{R}^d$ with intensity function $\lambda(x|\Phi_\mathrm{p}) = \bar{m}\lambda_\mathrm{p}\sum_{v\in\Phi_\mathrm{p}} f(x-v)$.

Coverage Analysis when BSs are PCP: Challenges



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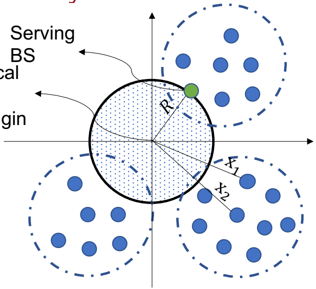
- First, condition on the parent PPP of BS PCP, derive conditional coverage: $\mathtt{P_c}|\Phi_{\rm p}$.
- Finally, decondition $\mathtt{P_c}|\Phi_{\rm p}$ w.r.t. the distribution of $\Phi_{\rm p}$.

Coverage Analysis when BSs are PCP: Challenges



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- First, condition on the parent PPP of BS PCP, derive conditional coverage: $\mathtt{P_c}|\Phi_{\rm p}$.
- Finally, decondition $\mathtt{P_c}|\Phi_{\rm p}$ w.r.t. the distribution of $\Phi_{\rm p}$.

It is important that $\mathtt{P_c}|\Phi_{\rm p}$ is a standard functional of $\Phi_{\rm p}$ (such as PGFL, SPFL etc).

New Spatial Models of Cellular Networks: PCP meets PPP

The general K-tier HetNet model:

- Two sets of BS PPs:
  - K_1 BS tiers are modeled as independent PPPs. The index set: $\mathcal{K}_1$.
  - K_2 BS tiers are modeled as independent PCPs. The index set: $\mathcal{K}_2$.
- User Distribution:
  - Type1: Users can be independent PPP.
  - The BSs of the q-th tier (Φ_q) are coupled with the user locations.
    - Type2: Users are PCP with BSs at the cluster center $(q \in \mathcal{K}_1)$
    - Type3: Users and BSs are PCP with same $(q \in \mathcal{K}_2)$

Models 1-4 from the previous slide are special cases of this *general* model.

How to handle the Spatial Coupling?

The typical user, under spatial coupling will observe the Palm version of $\Phi^{(q)}$.

Isolate the spatial coupling by define a virtual 0-th tier Φ_0.

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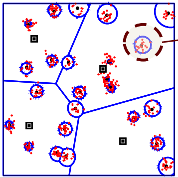
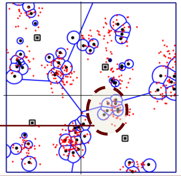


Type 2 users

If users form a PCP around BS locations, the 0^{th} tier is a BS at the cluster center of the typical user.

If both users and BSs are modeled as (coupled) PCPs, the 0^{th} tier is the set of BSs belonging to the same cluster center of the typical user.

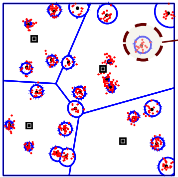


Type 3 users

How to handle the Spatial Coupling?

The typical user, under spatial coupling will observe the Palm version of $\Phi^{(q)}$.

Isolate the spatial coupling by define a virtual 0-th tier Φ_0.



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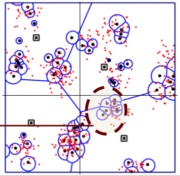


Type 3 users

$$\Phi^{(0)} := \begin{cases} \varnothing; & \text{for Type 1 users,} \\ \{z_o\}; & \text{for Type 2 users,} \\ z_o + \mathcal{B}^{(q)}; & \text{for Type 3 users.} \end{cases}$$

Max Average Received Power based Association

Cell Association: if x^* is the location of the BS serving the typical user,

$$x^* = \arg\max_{\{\widetilde{x}_k, k \in \mathcal{K}\}} P_k \|\widetilde{x}_k\|^{-\alpha},$$

where $\widetilde{x}_k = \arg\max_{x \in \Phi^{(k)}} P_k \|x_k\|^{-\alpha} = \arg\min_{\{x \in \Phi^{(k)}\}} \|x\|$ is the location of candidate serving BS in $\Phi^{(k)}$.

Next Steps

- Construct Φ_0 to handle user BS coupling.
- If $\mathcal{S}_k = \mathbf{1}(x^* \in \Phi^{(k)})$ denotes k-th tier association event,

$$\mathtt{P_c} = \mathbb{P}\Bigg(\bigcup_{k \in \mathcal{K}_1 \cup \mathcal{K}_2 \cup \{0\}} \{\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k\}\Bigg) = \sum_{k \in \mathcal{K}_1 \cup \mathcal{K}_2 \cup \{0\}} \mathbb{P}\Big(\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k\Big),$$

Coverage Analysis: Max Avg. Power

- Conditioning step: Condition on the locations of all the parent points of $\Phi^{(i)}, \forall\ i \in \mathcal{K}_2$. Derive conditional coverage probability:

$$\mathtt{P_c}^{\rm cond}\big(\cup_{i\in\mathcal{K}_2'} \Phi_{\rm p}^{(i)}\big) := \sum_{k\in\mathcal{K}_1\cup\mathcal{K}_2\cup\{0\}} \underbrace{\mathbb{P}\bigg(\{\mathtt{SIR}(x^*) > \beta_k, \mathcal{S}_k\}\bigg|\Phi_{\rm p}^{(i)}, \forall i \in \mathcal{K}_2'\bigg)}_{\text{Per-tier coverage}, \mathtt{P_c}_k}.$$

  Need to have some structural properties (next slide).
- Deconditioning step: Decondition the conditional coverage w.r.t. the distributions of $\Phi_{\rm p}^{(i)},\ \forall\ i \in \mathcal{K}_2$. Thus, $\mathtt{P_c} = \mathbb{E}\big[\mathtt{P_c}^{\rm cond}\big(\cup_{i\in\mathcal{K}_2'} \Phi_{\rm p}^{(i)}\big)\big]$.

Coverage Analysis: Max Avg. Power

We are able to express coverage as a product of PGFLs and SPFLs of the BS PPPs (for $k \in \mathcal{K}_1$) or parent PPPs of BS PCPs (for $k \in \mathcal{K}_2$).

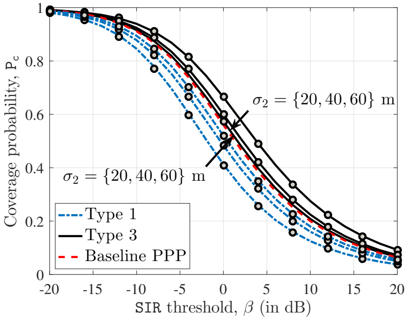
One representative result for Type 1 and 3 users, when $k \in \mathcal{K}_2 \cup \{0\} = \mathcal{K}_2'$,

$$\mathtt{P_c}_k = \bar{m}_k \int_0^\infty \overbrace{\prod_{j_1 \in \mathcal{K}_1} \mathbb{E}\Big[v_{j_1,k}(r)\Big]}^{\text{PGFL of } \Phi^{(j_1)}} \prod_{j_2 \in \mathcal{K}_2' \setminus \{k\}} \overbrace{\mathbb{E}\Big[\prod_{z \in \Phi_{\mathrm{p}}^{(j_2)}} \mathcal{C}_{j_2,k}(r, \|z\|)\Big]}^{\text{PGFL of } \Phi^{(j_2)}}$$

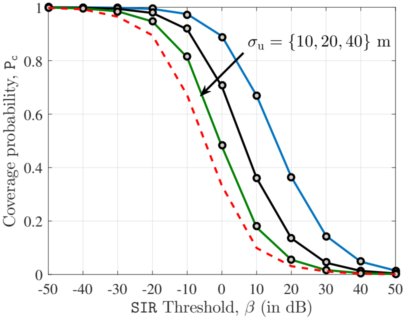
$$\times \underbrace{\mathbb{E}\Bigg[\bigg(\sum_{z \in \Phi_{\mathrm{p}}^{(k)}} f_{\mathrm{d}_k}(r|z) \prod_{z \in \Phi_{\mathrm{p}}^{(k)}} \mathcal{C}_{k,k}(r, z)\bigg)\Bigg]}_{\text{SPFL of } \Phi^{(k)}} \mathrm{d}r.$$

$v_{j_1,k}(r) = \exp\left(-\pi r^2 \lambda_{j_1} \bar{P}^2_{j_1,k} \rho(\beta_k, \alpha)\right)$. See [Saha2019] for the definitions of $\rho(\cdot)$ and $\mathcal{C}_{ij}(\cdot,\cdot)$.

Results



$\mathtt{P_c}$ vs. SIR threshold for Type 1 and 3 users ($\alpha = 4$, $P_2 = 10^3 P_1$, $\lambda_1 = 1$ km^{-2}, $\lambda_{p2} = 25$ km^{-2}, and $\bar{m}_2 = 4$).



$\mathtt{P_c}$ vs SIR threshold for Type 2 users ($\alpha = 4$, $P_1 = 10^3 P_2$, $\lambda_2 = 100\lambda_1 = 100/\pi(0.5$ km$)^2$).

- Coverage is strongly dependent on cluster size (measure of spatial coupling).
- As cluster size increases, coverage converges to $\mathtt{P_{c\,PPP}}$ (next slide).

Convergence to the baseline PPP Model

Proposition 1 (Weak Convergence of PCP to PPP)

If $\Phi^{(k)}$ is a PCP with parameters $(\lambda_{\mathrm{p}_k}, f_{k,\xi}, \bar{m}_k)$ then

$$\Phi_k \rightarrow \bar{\Phi}_k \ (weakly) \text{ as } \xi \rightarrow \infty,$$

where $\bar{\Phi}_k$ is a PPP of intensity $\bar{m}_k \lambda_{\mathrm{p}_k}$ if $\sup(f_k) < \infty$.

Proposition 2

The limiting PPP $\bar{\Phi}^{(k)}$ and the parent PPP $\Phi_{\mathrm{p}}^{(k)}$ of $\Phi^{(k)}$ $(k \in \mathcal{K}_2)$ are independent, i.e.,

$$\lim_{\xi \to \infty} \mathbb{P}\big(\Phi_k(A_1) = 0, \Phi_{\mathrm{p}_k}(A_2) = 0\big) = \mathbb{P}\big(\bar{\Phi}_k(A_1) = 0)\mathbb{P}(\Phi_{\mathrm{p}_k}(A_2) = 0\big),$$

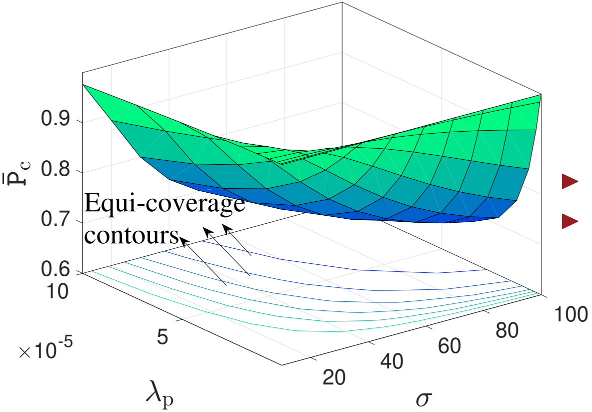
where $A_1, A_2 \subset \mathbb{R}^2$ are arbitrary closed sets.

See [Saha2018] for more details. These results have recurred in all our works on PCP.

Equi-Coverage Contours

For the general HetNet model, it is possible to find *equi-coverage contours*) in the parameter space[¶].

For a single tier network with PCP distributed BSs, the contours are:



- TCP: $\lambda_p \sigma^2 =$ constant.
- MCP: $\lambda_p \mathtt{R}^2 =$ constant.

[¶] Afshang, Saha, Dhillon, "Equi-Coverage Contours in Cellular Networks", *IEEE Wireless Commun. Letters*, 2018.

SIR Meta Distribution

Definition

The meta distribution of SIR is the CCDF of the link reliability $P_s(\beta) \triangleq \mathbb{P}(\mathtt{SIR} > \beta|\Phi)$, i.e., $\bar{F}(\beta,\theta) = \mathbb{P}[P_s(\beta) > \theta], \beta \in \mathbb{R}^+, \theta \in (0,1]$.

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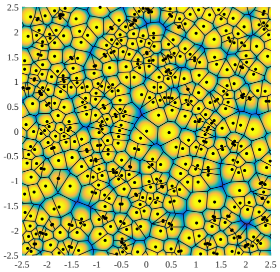
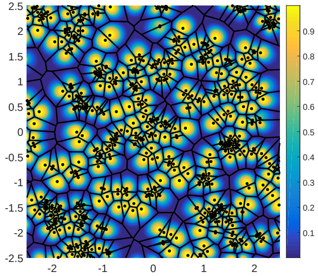
$$\underbrace{\mathbb{P}(\mathtt{SIR} > \beta|\Phi)}_{\text{Prob. that a user achieves SIR} > \beta}$$

$$\bar{F}(\beta,\theta) = \underbrace{\mathbb{P}(\mathbb{P}(\mathtt{SIR} > \beta|\Phi) > \theta)}_{\text{Fraction of users that achieve SIR} > \beta \text{ w.p. at least } \theta}$$



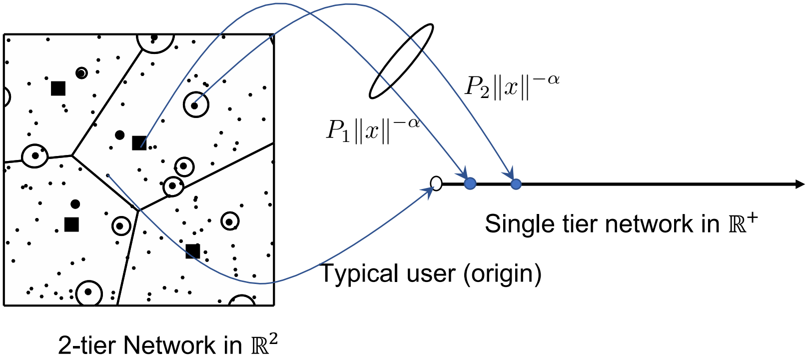
(a) Link reliability for PPP BSs



(b) Link reliability for PCP BSs

$$\bar{F}(\beta,\theta) = \lim_{r\to\infty} \frac{\sum_{u\in\Phi_{\rm u}\cap rB} \mathbf{1}(\mathbb{P}(\mathtt{SIR}(u) > \beta) > \theta)}{\mathbb{E}[\Phi_{\rm u}(rB)]}, \ \forall \text{ Borel set } B \subset \mathbb{R}^2.$$

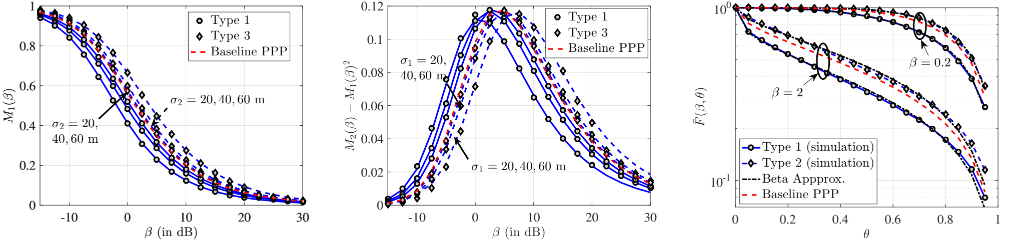
SIR Meta Distribution for General HetNet Model (1)



- Like coverage (which is $M_1(\beta)$), $M_b(\beta)$ is also expressed as the product of PGFLs and SPFLs of BS PPPs (when $i \in \mathcal{K}_1$) and parent PPPs of BS PCPs (when $i \in \mathcal{K}_2$)[‖].
- From the b-th moments, the CDF of meta distribution can be obtained by moment matching a beta kernel.

[‖] **[Saha2020a] C. Saha, M. Afshang, and H. S. Dhillon, "Meta Distribution of Downlink SIR in a Poisson Cluster Process-based HetNet Model"**, arxiv.org/abs/2007.05997.

SIR Meta Distribution for General HetNet Model (2)



(a) Mean of meta distribution. (b) Variance of meta distribution. (c) Meta distribution ($\sigma_2 = 40$ m)

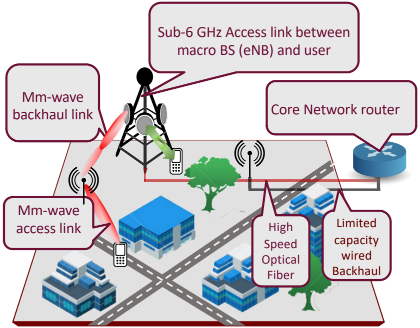
Meta distribution of SIR for Type 1 and Type 3 users in a two tier network. Details of the network configuration: $K = 2$, $\mathcal{K}_1 = \{1\}$, $\mathcal{K}_2 = \{2\}$, $q = 2$ for Type 3, $\alpha = 4$, $P_2 = 10^2 P_1$, $\lambda_{p_2} = 2.5$ km^{-2}, $\lambda_{p_1} = 1$ km^{-2}, $\bar{m}_2 = 4$, and $\sigma_2 = \sigma_u$. Markers indicate the values obtained from Monte Carlo simulations.

Section IV

Integrated Access and Backhaul Model

Modeling user hotspots in IAB | Rate Analysis

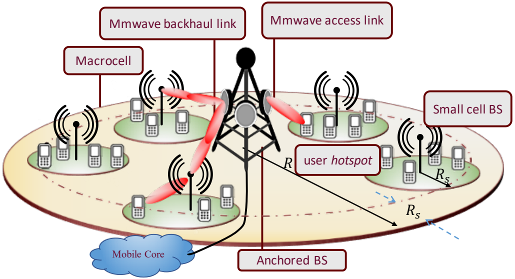
Integrated Access and Backhaul in 5G



- 5G requires Gbps backhaul capacity, which requires new spectrum (e.g. mm-wave) for both access and backhaul.
- Integrated Access-and-Backhaul (IAB): when the access and the backhaul links share the same wireless channel.
- Self-backhaul will become a driving technology owing to more ubiquitous deployment of small cells, e.g. in vehicles, trains, lamp-posts etc.
- Need to develop analytical framework for IAB-enabled cellular network to gain key design insights.

Modeling IAB with Stochastic Geometry

We used the idea of Type 2 users to construct a spatial model for IAB**.



IAB system model

- Considered a single microcell (circular) with n SBSs and $\bar{m}$ users per SBS.
- Blocking: Each link of distance r is LOS or NLOS with probability $p(r) = e^{-r/\mu}$.
- User Association: Users can connect to the SBS at the center of the hotspot or the MBS.

**C. Saha, M. Afshang, and H. S. Dhillon, "Bandwidth Partitioning and Downlink Analysis in Millimeter Wave Integrated Access and Backhaul for 5G", IEEE TWC, 2018.

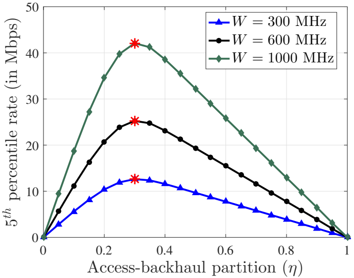
Resource Allocation in IAB

Total system BW W is split into two parts: $W_b = \eta W$ for backhaul and $W_a = (1-\eta)W$ for access. $\eta \in (0,1]$: access-backhaul split.

- Access BW reused at each BS, split equally among the load on that BS.
- Backhaul BW is shared among n SBSs by either of the following ways–
  - Equal Partition: $W_s(x) = \frac{W_b}{n}$.
  - Instantaneous Load-based Partition: $W_s(x) = \frac{N_x^{\rm SBS}}{N_x^{\rm SBS}+\sum_{i=1}^{n-1} N_{x_i}^{\rm SBS}} W_b$.
  - Average Load-based Partition: $W_s(x) = \frac{\bar{N}_x^{\rm SBS}}{\bar{N}_x^{\rm SBS}+\bar{N}_{x_i}^{\rm SBS}} W_b$

$N_x^{\rm SBS}$:= inst. load on SBS at x, $\bar{N}_x^{\rm SBS}$:= avg. load.

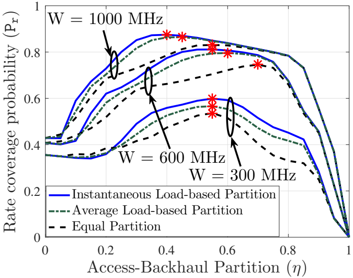
Key Insights (1)



5-th percentile rate for instantaneous load based partition ($n = 10$).

There exists an optimal access-backhaul partition for which rate is maximized.

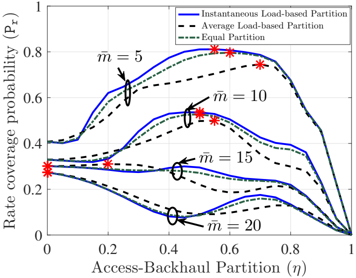
Key Insights (2)



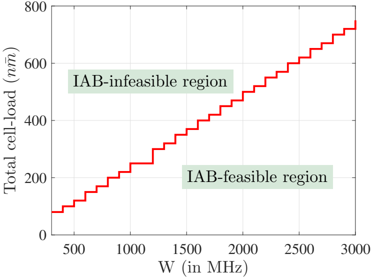
Comparison of different BW partition strategies (target rate 50 Mbps, $n = 10$).

In terms of maximum rate coverage, Inst. partition$>$ Avg. partition$>$ Equal partition.

Key Insights (3)



Rate coverage for different number of users per hotspot ($W = 600$ Mhz, $\rho = 50$ Mbps).



Total cell-load beyond which IAB-ebabled network is outperformed by a macro-only network.

Take-Aways

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- Proposed spatial models of HetNet based on PCP to capture the effects of spatial coupling.

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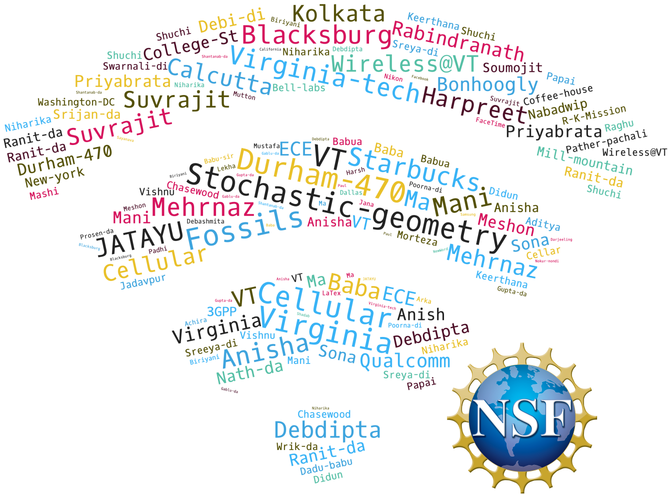
- Built fundamental mathematical tools for the coverage analysis of PCP-based models.
- Showed that network performance is strongly dependent on the extent of spatial coupling.
- Proposed spatial models of IAB networks. Characterized the rate distribution.
- Proposed determinantal point process (DPP) based network models which is amenable to learning the spatial distribution from a trianing set.

Acknowledgments

- Dr. Dhillon.
- Committee members.
- National Science Foundation (Grants CNS-1617896 and CNS- 1923807).
- Current and Past lab-mates.
- Friends and Family.



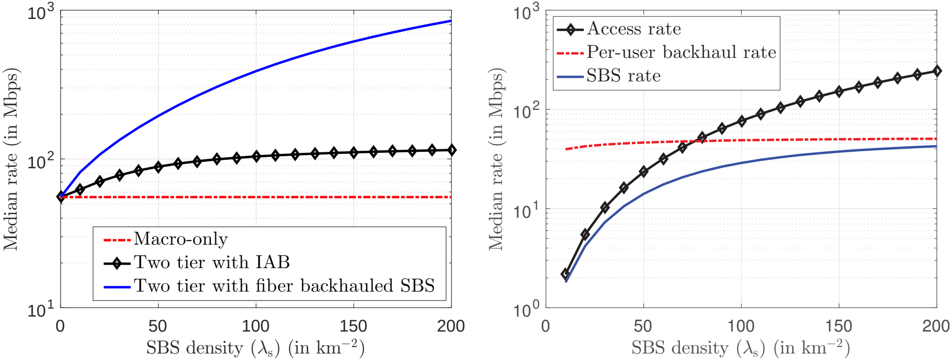
Thank You!

Appendix

Load Distribution

- Load on a typical BS is the number of users associated with that BS.
- The load on a typical BS is equal to the number of points of the user point process ($\Phi_{\rm u}$) falling in the typical Poisson Voronoi cell ($\mathcal{C}_0$) of Φ (assuming single tier and max-power based association).
- When Φ and $\Phi_{\rm u}$ are PPPs, the PMF of load is known. $\Phi_{\rm u}(\mathcal{C}_0) \sim {\rm Poisson}({\rm vol}(\mathcal{C}_0))$.
- When $\Phi_{\rm u}$ is PCP, we characterized the mean and variance of $\Phi_{\rm u}(\mathcal{C}_0)$ when $\Phi_{\rm u}$ is a PCP.
- We also derived tight approximation of the PMF of $\Phi_{\rm u}(\mathcal{C}_0)$.

Limits of Network Densification



For IAB, the SBS densification does not improve rate beyond some saturation limit.

Insights

- Developed an analytical model of mm-wave two-tier HetNet with IAB.
- Derived the downlink rate coverage of this network.
- Introduced reasonable assumptions to handle load distributions under correlated blockage.
- Bias factors do not play a prominent role in boosting data rate in IAB-enabled HetNets.
- The improvement in rate quickly saturates with SBS densification.

Future Works.

- Analysis of multi-hop IAB networks.
- Characterize the delays occurring in the IAB networks.

Contact and Nearest Distance Distributions of MCP

Theorem 1

If Φ is a MCP in $\mathbb{R}^d$, the CDF of its contact distance is given by

$$H_s(r) = 1 - \exp\Bigg(- c_d\lambda_{\rm p}\bigg((r+\mathtt{R})^d - |r-\mathtt{R}|^d \exp\bigg(- \frac{\bar{m}}{\mathtt{R}^d}(\min(r,\mathtt{R}))^d\bigg) - d\int_{|r-\mathtt{R}|}^{r+\mathtt{R}} \exp\left(-\frac{\bar{m}}{c_d\mathtt{R}^d}\Upsilon(r,\mathtt{R},y)\right) y^{d-1}{\rm d}y\bigg)\Bigg), \quad (1)$$

where $\Upsilon(r_1, r_2, t)$ the area of intersection of two discs of radii r_1, r_2 and distance between the centers t. The CDF of its nearest distance distance is given by

$$D(r) = \begin{cases} 1-(1-H_s(r))R^{-d}\bigg(\exp\bigg(-\frac{\bar{m}(\min(\mathtt{R},r))^d}{\mathtt{R}^d}\bigg)|\mathtt{R}-r|^d \\ +d\int_0^{\mathtt{R}}\exp\left(-\frac{\bar{m}}{c_d\mathtt{R}^d}\Upsilon(r,\mathtt{R},x)\right)x^{d-1}{\rm d}x\bigg), & \text{if } r\leq 2\mathtt{R}, \\ 1-(1-H_s(r))e^{-\bar{m}}, & \text{if } r > 2\mathtt{R}. \end{cases} \quad (2)$$

Contact and Nearest Distance Distributions of TCP

Theorem 2

If Φ is a TCP in $\mathbb{R}^d$, the CDF of its contact distance is given by

$$H_s(r) = 1 - \exp\left(-\lambda_{\rm p} 2^d c_d \int_0^\infty \left(1 - \exp\big(- \bar{m}(1 - Q_{\frac{d}{2}}(\sigma^{-1}y, \sigma^{-1}r))\big)\right) y^{d-1} \mathrm{d}y\right), \quad (3)$$

where $Q_{\frac{d}{2}}$ is the Marcum-Q function. The CDF of the nearest neighbor distance is given by

$$D(r) = 1 - (1 - H_s(r)) \int_0^\infty \exp\left(- \bar{m}(1 - Q_{\frac{d}{2}}(\sigma^{-1}y, \sigma^{-1}r))\right) \chi_d(\sigma^{-1}y)\, \mathrm{d}y, \quad (4)$$

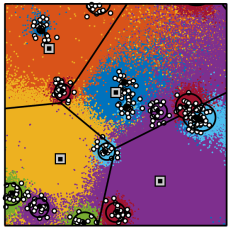
where $\chi_d(\cdot)$ is the PDF of a Chi distribution with d degrees of freedom.

Max Instantaneous Power based Association

SIR of the link between the typical user and a BS at $x \in \Phi^{(k)}$:

$$\mathtt{SIR}(x) = \frac{P_k h_x \|x\|^{-\alpha}}{\underbrace{\mathcal{I}(\Phi^{(k)} \setminus \{x\}) + \sum_{j \in \mathcal{K}\setminus\{k\}} \mathcal{I}(\Phi^{(j)})}_{\mathcal{I}(\Phi^{(i)}) = \sum_{y\in\Phi^{(i)}} P_i h_y \|y\|^{-\alpha} \text{:aggregate interference}}}. \quad (5)$$

Cell Association: If x^* denotes the location of the BS serving the typical user at origin,



$$\max_{x_k\in\Phi^{(k)}} P_k h_k \|x_k\|^{-\alpha} = \arg \max_{x \in \cup_{k\in\{\mathcal{K}_1 \cup \mathcal{K}_2\}} \Phi^{(k)}} \mathtt{SIR}(x).$$

- Since small scale fading is part of the association rule, the cells are *irregular* shaped.
- This strategy is *coverage-optimal*.
- Lends to a nice structure of analysis (next slide).

Downlink Coverage

Definition (Coverage probability)

Assuming β is the SIR-threshold, coverage probability is defined as:

$$\mathtt{P_c} = \mathbb{P}(\mathtt{SIR}(x^*) > \beta | o \in \Phi_{\mathrm{u}}) = \mathbb{P}\left(\max_{\substack{x \in \Phi^{(k)}, \\ k \in \mathcal{K}_1 \cup \mathcal{K}_2}} \mathtt{SIR}(x) > \beta \middle| o \in \Phi_{\mathrm{u}} \right)$$

$$= \mathbb{E}^{!o}_{\Phi_{\mathrm{u}}}\left[\mathbf{1}\left(\bigcup_{k \in \mathcal{K}_1 \cup \mathcal{K}_2} \bigcup_{x \in \Phi^{(k)}} \{\mathtt{SIR}(x) > \beta\} \right) \right].$$

Assuming $\beta > 1$, $\mathtt{P_c} = \sum_{k \in \mathcal{K}} \mathtt{P_c}_k$, where $\mathtt{P_c}_k$ = per-tier coverage [Dhillon2012].
Under i.i.d. Rayleigh fading assumption,

$$\mathtt{P_c}_k = \underbrace{\mathbb{E}\left[\sum_{x \in \Phi^{(k)}} \prod_{y \in \Phi^{(k)} \setminus \{x\}} \mu_{k,k}(x,y) \right]}_{\text{Sum-product functional (SPFL)}} \times \prod_{k_1 \in \mathcal{K}_1 \setminus \{k\}} \overbrace{\mathbb{E}\left[\prod_{y \in \Phi^{(k_1)}} \mu_{k_1,k}(x,y) \right]}^{\text{Probability generating functional (PGFL)}},$$

where $\mu_{i,j}(x,y) = \frac{1}{1 + \frac{\beta P_i \|x\|^\alpha}{P_j \|y\|^\alpha}}$.

Sum-product functional

Definition

If Φ is a point process in $\mathbb{R}^d$, the SPFL of Φ is $S_\Phi[g,v] := \mathbb{E}\left[\sum_{x\in\Phi} g(x) \prod_{y\in\Phi} v(x,y)\right]$, where $v : \mathbb{R}^d \times \mathbb{R}^d \to (0,1]$ and $g : \mathbb{R}^d \to \mathbb{R}^+$ are measurable.

Lemma 1

The expressions of reduced SPFL for different PPs are given as: $S^!_\Phi[g,v] =$

$$\int_{\mathbb{R}^d} g(x) \underbrace{G_\Phi[v(x,\cdot)]}_{\text{PGFL of } \Phi} \Lambda(\mathrm{d}x)$$ Φ is PPP with intensity measure Λ,

$$\bar{m}\lambda_{\mathrm{p}} \int_{\mathbb{R}^d} g(x) G_\Phi[v(x,\cdot)] G_{\mathcal{B}^!_o}[v(x,\cdot)] \mathrm{d}x$$ Φ is PCP with parameters $(\lambda_{\mathrm{p}}, \bar{m}, f)$,

$$\int_{\mathbb{R}^2} \int_{\mathbb{R}^2} g(x) G_{\Phi^{(0)}|z_o}[v(x,\cdot)] \times \left(\bar{m}_q \int_{\mathbb{R}^2} v(x,y) f(y|z_o) \mathrm{d}y + 1\right) f(x|z_o) f_{\mathrm{u}}(z_o)\, \mathrm{d}x\, \mathrm{d}z_o,$$ Type 3 users, $\Phi^{(0)}$.

Coverage Probability: One Representative Result

Theorem 3 (Coverage Probability: Type 2 Users)

$\mathrm{P_c}$ of a typical user for $\beta > 1$ is –

$$\mathrm{P_c} = \sum_{k \in \mathcal{K}} \mathrm{P_c}_k,$$

where $\mathrm{P_c}_k$ is the per-tier coverage:

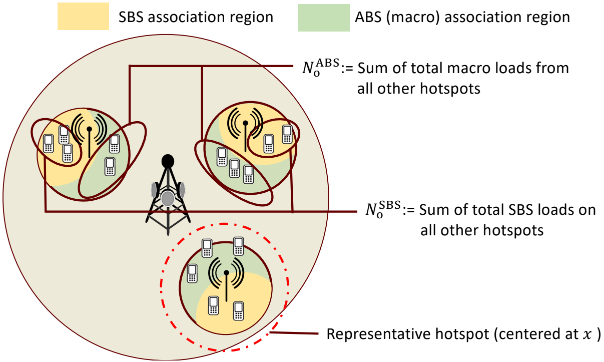
$$k = 0 : \mathrm{P_c}_k = \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(z_o, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(z_o, \cdot)] f_{\mathrm{u}}(z_0) \mathrm{d}z o,$$

$$k \in \mathcal{K}_1 : \mathrm{P_c}_k = \lambda_k \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(x, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(x, \cdot)] G_{\Phi^{(0)}}[\mu_{q,k}(x, \cdot)] \, \mathrm{d}x,$$

$$k \in \mathcal{K}_2 : \mathrm{P_c}_k = \bar{m}_k \lambda_{\mathrm{p}_k} \int_{\mathbb{R}^2} \prod_{k_1 \in \mathcal{K}_1} G_{\Phi^{(k_1)}}[\mu_{k_1,k}(x, \cdot)] \prod_{k_2 \in \mathcal{K}_2} G_{\Phi^{(k_2)}}[\mu_{k_2,k}(x, \cdot)]]$$

$$\times G_{\mathcal{B}_o^{(k)!}}[\mu_{k,k}(x, \cdot)] G_{\Phi^{(0)}}[\mu_{q,k}(x, \cdot)] \mathrm{d}x.$$

Association Regions and Load



Contributions

- Derived the distribution of loads on different BSs.
- Derived the rate coverage probability for different BW partitioning strategies.